

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 3443

SHEARING EFFECTIVENESS OF INTEGRAL STIFFENING

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Washington

June 1955

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SUMMARY

Values of coefficients for defining the effectiveness of integral stiffeners in resisting shear deformations of the plate of which they are an integral part are presented for a wide range of proportions of rectangular stiffeners with circular fillets. The coefficients are evaluated by the use of an electrical analog computer. Formulas are given in which these coefficients may be employed to calculate the elastic constants associated with the twisting and shearing of integrally stiffened plates, either directly, as in the case of simple longitudinal or transverse stiffening, or through the intermediate evaluation of previously defined shearing-effectiveness parameters, as in the case of more complicated stiffener patterns. The fillet radius is shown to contribute appreciably to the degree of penetration of the stresses from the skin into the stiffeners. Thus, through the use of suitable combinations of rib proportions and fillet radii, simple longitudinal or transverse integral stiffening can be made to contribute to the overall shear stiffness of the plate-stiffener combination.

INTRODUCTION

The effectiveness of integrally stiffened plates, as demonstrated in references 1 to 4, is in part due to the fact that the stresses in the skin of the plate are conducted into the integral stiffeners. Even under simple loadings, a complicated stress distribution within the cross section is produced, and in consequence the evaluation of the elastic constants for the plate-stiffener combination is difficult.

Formulas for the thirteen elastic constants of plates with integral stiffeners and a method for analytically obtaining upper and lower limits on the parameters of the formulas are presented in reference 4. In some cases the differences between the constants calculated by the upper- and lower-limit assumptions are substantial. For example, upper-limit shear stiffnesses calculated in reference 4 were from 12 percent to 32 percent greater than the calculated lower-limit stiffnesses.

In the present paper a more refined analysis is made of the shearing effectiveness of integral stiffening. The approach used is that of imposing a quasi-shear deformation upon a repeating element of a plate with simple longitudinal or transverse stiffening (see fig. 1) and then solving the equations of elasticity associated with the imposed deformation. The method of solution is similar to the method used in solving the torsion problem of pages 258 to 263 of reference 5. This approach requires the solution of Laplace's equation over the cross section of the repeating element as shown in figure 2. Solutions for a wide range of proportions of rectangular stiffeners with circular fillets were obtained with a General Electric Analog Field Plotter (ref. 6) which was modified by the NACA to suit the needs of this particular problem. This modified field plotter is similar in operation and principle to an electrical analog computer described in reference 7.

Results of this analysis are presented in the form of tabulations and curves giving coefficients from which the shearing effectiveness of the integral stiffener may be evaluated. These coefficients may be used with the formulas of reference 4 for the calculation of the plate elastic constants; if the shearing stiffness of a plate with simple longitudinal or transverse stiffening is required, however, it may be determined more directly from the given coefficients through the use of formulas presented herein.

SYMBOLS

Plate Dimensions

A_W	area of perpendicular cross section of rib, sq in.
b_S	length of repeating element of integrally stiffened plate, in.
b_W	height of rib above plate, in.
H	total height of rib and plate, $t_S + b_W$, in.
h	z-distance from y-axis to boundary $\overline{cdef}$ of specimen (fig. 2)
l_S	length of plate between fillet and end of repeating element, $\frac{b_S - 2r_W - t_W}{2}$, in.
l_W	height of rib above fillet radius, $b_W - r_W$, in.

r_W	radius of fillet, in.
t_S	thickness of skin or plate, in.
t_W	thickness of rib or web, in.

Forces and Elastic Constants

C_k	coupling elastic constant associated with coupling between twist and shear and defined by equations (1) and (2), in.
D_k	twisting stiffness relative to x- and y-directions defined by equation (1), in-lb
D_{xy}	twisting stiffness relative to x- and y-directions defined by equation (3), in-lb
G	shear modulus of material, psi
G_k	shear stiffness of plate in xy-plane defined by equation (2), lb/in.
G_{xy}	shear stiffness of plate in xy-plane defined by equation (4), lb/in.
K	torsion constant defined by equation (27)
K'	torsion constant for sections shown in figure 6(a)
M_{xy}	intensity of resultant twisting torque, lb
N_{xy}	intensity of resultant shearing force acting in plane $z = t_S/2$, lb/in.
T	coupling elastic constant associated with coupling between twist and shear defined by equations (3) and (4), lb^{-1}

Special Symbols Used in Shearing-Effectiveness Analysis

c	coupling coefficient defined by equation (A30)
$c_{T_{xy}}$	coupling coefficient defined by equation (A25)

$c_{T_{yx}}$	coupling coefficient defined by equation (A26)
F_{xy}	intensity of resultant shearing force acting in y-direction in plane $z = t_S/2$, lb/in.
F_{yx}	intensity of resultant shearing force acting in x-direction in plane $z = t_S/2$, lb/in.
j	coefficient of twisting stiffness defined by equation (6)
P	boundary value of stress function or boundary value of electrical potential field (fig. 2)
q	coefficient of shearing stiffness defined by equation (A22) and determined by equation (A24)
S_1	integral of stress function defined by equation (A35)
S_2	integral of stress function defined by equation (A36)
T_{xy}	intensity of resultant twisting torque acting on planes perpendicular to x-axis, lb
T_{yx}	intensity of resultant twisting torque acting on planes perpendicular to y-axis, lb
α'	coefficient used in reference 4 to locate effective centroid of part of rib for resisting twisting deformation
β'	coefficient used in reference 4 to define effectiveness of rib in resisting shear deformation
β	coefficient used in reference 4 to define effectiveness of rib in resisting stretching in its transverse direction
δ	magnitude of pure shear distortion imposed upon repeating element of integrally stiffened plate (fig. 3), in.

General Symbols

u, v, w	displacements in x-, y-, and z-directions, respectively, in.
x, y, z	orthogonal coordinates; z measured normal to plane of plate, and x and y measured in plane of plate

$\gamma_{xy}, \gamma_{xz}, \gamma_{yz}$	components of shear strain
$\bar{\gamma}_{xy}$	average γ_{xy} over length of repeating element defined by equation (A21)
$\epsilon_x, \epsilon_y, \epsilon_z$	components of normal strain
$\sigma_x, \sigma_y, \sigma_z$	components of normal stress, psi
$\tau_{xy}, \tau_{xz}, \tau_{yz}$	components of shear stress, psi
ξ, η	transformed orthogonal coordinate system defined in equations (22)
ϕ	stress function defined by equations (A6)
ψ	warping function defined by equation (A1)

STATEMENT OF PROBLEM

The force-distortion relationships for the twisting and shearing of rectangular orthotropic integrally stiffened plates having their axes of principal stiffness parallel or perpendicular to the sides of the plate (figs. 1 and 2) may be written, as noted in reference 4, in two forms as follows:

$$M_{xy} = 2D_k \frac{\partial^2 w}{\partial x \partial y} + C_k N_{xy} \quad (1)$$

$$\gamma_{xy} = -2C_k \frac{\partial^2 w}{\partial x \partial y} + \frac{N_{xy}}{G_k} \quad (2)$$

or

$$\frac{\partial^2 w}{\partial x \partial y} = \frac{M_{xy}}{D_{xy}} + T N_{xy} \quad (3)$$

$$\gamma_{xy} = 2T M_{xy} + \frac{N_{xy}}{G_{xy}} \quad (4)$$

in which the elastic constants D_{xy} , G_{xy} , and T are related to D_k , G_k , and C_k by the following formulas:

$$\left. \begin{aligned} D_{xy} &= 2D_k \\ G_{xy} &= \frac{G_k D_k}{D_k + C_k^2 G_k} \\ T &= -\frac{C_k}{2D_k} \end{aligned} \right\} \quad (5)$$

These elastic constants can be evaluated in terms of the three coefficients j , q , and c which express the effectiveness of the stiffeners in resistance to twisting, to shearing, and to coupling between twisting and shearing, respectively. For plates with simple longitudinal or transverse integral stiffeners, the equations for the elastic constants in terms of these coefficients are determined as follows:

$$D_k = \frac{1}{2} G j t_S^3 \quad (6)$$

$$G_k = G q t_S \quad (7)$$

$$C_k = -c t_S \quad (8)$$

$$D_{xy} = G j t_S^3 \quad (9)$$

$$G_{xy} = \frac{G j q t_S}{2c^2 q + j} \quad (10)$$

$$T = \frac{c}{G j t_S^2} \quad (11)$$

The determination of the coefficient j is presented subsequently in the text and that of the coefficients q and c , in the appendix.

Expressions for these elastic constants have also been derived in reference 4 for plates with stiffeners in a variety of patterns, where the effectiveness of the stiffeners for resisting twisting and shearing is expressed in terms of two parameters α' and β' . A method for the evaluation of α' was given in reference 4, based on the work of reference 8, but no basis for the evaluation of β' was then available. (The suggestion was made in reference 4 that β' be assumed equal to β . Subsequent experimentation has shown that for a wide range of proportions, $\beta = 7/8 \beta'$.) Values of α' and β' may now be determined, however, in terms of j and q ; thus

$$\alpha' = \frac{t_S}{H} \sqrt{\frac{1}{2} \left(\frac{q}{q-1} \right) \left(j - \frac{1}{6} \right)} \quad (12)$$

$$\beta' = \frac{q-1}{A_W/b_S t_S} \quad (13)$$

where

$$\frac{A_W}{b_S t_S} = \frac{t_S}{b_S} \left[\frac{b_W}{t_S} \frac{t_W}{t_S} + 2 \left(1 - \frac{\pi}{4} \right) \left(\frac{r_W}{t_S} \right)^2 \right] \quad (14)$$

Equations (12) and (13) are derived in the appendix.

The problem considered in the present paper is the evaluation of j , q , and c . Actual values of these coefficients are obtained herein for only rectangular stiffeners with circular fillets, but the methods of analysis are applicable to stiffeners of any cross section.

PROCEDURES FOR DETERMINATION OF COEFFICIENTS

Determination of j

References 8 and 9 give torsion constants for sections such as those illustrated in figures 1 and 2. The coefficient j used in the evaluation of the elastic constants of integrally stiffened plates is related to the conventional torsion constant K as follows:

$$j = \frac{K}{2 \frac{b_S}{t_S} t_S^4} \quad (15)$$

Check tests of the twisting stiffness of a few plates of different proportions have been made which confirm the results of references 8 and 9. No further analysis of the twisting stiffness of integrally stiffened plates was therefore made. Further information on the evaluation of j is given in the section entitled "Results and Applications."

Determination of q and c

A quasi-shear deformation was imposed on the repeating element of the integrally stiffened plate as shown in figure 3, and the stress resultants required to produce this deformation (fig. 4) were found. The details of this analysis are presented in the appendix and the resulting equations are found to be

$$N_{xy} = qGt_S \bar{\gamma}_{xy} \quad (16)$$

and

$$M_{xy} = -ct_S N_{xy} \quad (17)$$

where q and c are given by

$$q = \frac{b_S/t_S}{2 \frac{l_S}{t_S} + S_1} \quad (18)$$

$$c = \frac{\frac{r_W}{t_S} + \frac{1}{2} \frac{t_W}{t_S} + \frac{b_W}{t_S} \frac{t_W}{t_S} + 2 \left(1 - \frac{\pi}{4}\right) \left(\frac{r_W}{t_S}\right)^2 - S_2 - \frac{1}{2} S_1}{b_S/t_S} \quad (19)$$

in which b_S , b_W , l_S , r_W , t_S , and t_W are dimensions of the plate (see fig. 2), and S_1 and S_2 are integrals of a stress function ϕ defined in the appendix. The integrals are

$$S_1 = \frac{1}{Pt_S} \int_{-\left(r_W + \frac{t_W}{2}\right)}^{r_W + \frac{t_W}{2}} \phi_{z=t_S} dy$$

and

$$S_2 = \frac{1}{Pt_S} \int_{-(r_W + \frac{t_W}{2})}^{r_W + \frac{t_W}{2}} \int_{t_S}^h \phi \, dz \, dy$$

In order to evaluate these integrals, use was made of an electrical analogy. As is shown in the appendix, the stress function ϕ must satisfy Laplace's equation,

$$\frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad (20)$$

The application of Ohm's law to a thin conducting sheet of material (see ref. 7) shows that a function V , which describes an electrical potential field in the sheet, must satisfy Laplace's equation,

$$\frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 \quad (21)$$

Accordingly, a potential field was set up in a conducting sheet over a shape related linearly to the shape of the cross section under consideration (see fig. 2) by electrically duplicating the boundary conditions on the stress function. A self-balancing potentiometer was used to measure values of the potential over the conducting sheet as shown in figure 5. The desired integrals S_1 and S_2 were then computed by numerically integrating the potential readings over the cross section.

The potentiometer used was a modified General Electric Analog Field Plotter (ref. 6), and the conducting sheet was Type L Teledeltos paper. This paper was sufficiently homogeneous to give good results with the large-scale cross sections used, but an adjustment was required to take into account a 7-percent deviation from the maximum resistance which existed between the directions of principal resistance. This directional property was corrected for by distorting the figure according to the transformation

$$\left. \begin{aligned} z &= \xi \\ y &= \eta \sqrt{\frac{R_z}{R_y}} \end{aligned} \right\} \quad (22)$$

where R_y and R_z are electrical resistances in the y- and z-directions, respectively. The actual potential field in the sheet is described by

$$\frac{\partial^2 V}{\partial z^2} + \frac{R_z}{R_y} \frac{\partial^2 V}{\partial y^2} = 0 \quad (23)$$

Thus, a potential value measured from the distorted figure is equivalent to a potential at a corresponding point in an undistorted figure on uniform, nondirectional conducting paper.

As a check on the overall accuracy of the procedure, values of the integrals were calculated for a typical cross section by the iterative procedure of reference 10 and compared with those measured with the electrical analog. The difference between the two values was less than 1 percent.

RESULTS AND APPLICATIONS

The values of j , q , and c may be used directly in equations (6) to (11) for the calculation of the elastic constants associated with the twisting and shearing of plates with simple longitudinal or transverse integral stiffening. For plates having combined longitudinal and transverse or symmetrically skewed ribbing, values of j and q may be used in equations (12) and (13) to calculate corresponding value of the parameters α' and β' for use in the elastic-constant formulas of reference 4.

Evaluation of Coefficients

Evaluation of j .— Check tests performed in the Langley structures research laboratory in conjunction with this investigation have shown that the method of reference 8 gives accurate results only when $r_W/t_S \leq 1$ and $t_W/t_S \leq 2$. The following formulas, which can be derived by use of reference 8, should therefore be used only when $r_W/t_S \leq 1$ and $t_W/t_S \leq 2$:

$$j = \frac{1}{6} + \frac{1}{2} \frac{t_S}{b_S} \left[\frac{1}{3} \frac{b_W}{t_S} \left(\frac{t_W}{t_S} \right)^3 - 0.105 \left(\frac{t_W}{t_S} \right)^4 + a \left(\frac{d}{t_S} \right)^4 \right] \quad (24)$$

where

$$a = 0.094 + 0.070 \frac{r_W}{t_S} \quad (25)$$

and

$$\frac{d}{t_S} = \frac{\left(1 + \frac{r_W}{t_S}\right)^2 + \frac{t_W}{t_S} \left(\frac{r_W}{t_S} + \frac{1}{4} \frac{t_W}{t_S}\right)}{2 \frac{r_W}{t_S} + 1} \quad (26)$$

The results of reference 9 were found to agree closely with the check tests previously mentioned. Reference 9 presents torsional constants (designated herein as K') for the configuration shown in figure 6(a), in which $l_S = t_S$ and $l_W = t_W$. The results of references 8 and 9 and extrapolations of each of these sources are presented graphically in figure 6(b) for integrally stiffened plates of the proportions shown in figure 6(a). The extrapolations were partly guided by the experimental data from the check tests.

Inasmuch as the curves of figure 6(b) apply to only the configuration shown in figure 6(a), the torsional stiffness of any additional skin or rib height must be accounted for separately. Since the additional skin or rib is remote from the juncture, the stress distribution at the juncture will not significantly affect the distribution in the remote portions of the section; therefore, the additional torsional stiffness is approximately that of a rectangular section. The torsion constant K for any section is then the sum of K' (the torsion constant of the section shown in fig. 6(a)) and the torsion constants of the rectangular sections; thus

$$K = K' + \frac{1}{3} \left(2 \frac{l_S}{t_S} - 2\right) t_S^4 + \frac{1}{3} \left(\frac{l_W}{t_S} - \frac{t_W}{t_S}\right) \left(\frac{t_W}{t_S}\right)^3 t_S^4 \quad (27)$$

The relation between K and j is given in equation (15). When experimental data are available, j may be computed directly as

$$j = \frac{M_{xy}}{G t_S^3 \frac{\partial^2 w}{\partial x \partial y}} \quad (28)$$

Evaluation of q and c .— The values of the shearing and coupling effectiveness coefficients q and c determined by the electrical analogy are given in table I for a wide range of proportions of rectangular ribs with circular fillets. Also included in table I are the corresponding values of the parameter β' of reference 4.

Values of q and c are presented in figures 7 and 8 as plots of $\frac{b_S/t_S}{q} - 2 \frac{l_S}{t_S}$ and $\frac{b_S}{t_S} c$ against $\frac{r_W}{t_S}$. The curves apply only when the values of both b_S/t_S and b_W/t_S are equal to or greater than the largest values of these two parameters which appear in the table for the given values of r_W/t_S and t_W/t_S under consideration. These plots may be made because, beyond certain limits (the maximum values of b_S/t_S and b_W/t_S appearing in table I for the r_W/t_S and t_W/t_S under consideration), additional rib height or additional plate length between ribs will not affect the stress distribution at the juncture of the skin and rib.

Interpolation may be made by cross-plotting when a set of dimension ratios fall within the range of values presented in the table.

When b_S/t_S is found within the table but b_W/t_S lies beyond the range of the table, the values of q and c appearing under the largest value of b_W/t_S for the b_S/t_S being considered may be used. However, the value of β' must then be computed from equations (13) and (14) by using the actual value of b_W/t_S . When b_W/t_S is found within the table but b_S/t_S lies beyond the range of the table, the value of q for the largest value of b_S/t_S for the value of b_W/t_S being considered is used in equation (18) to obtain a value of S_1 . The value of b_S/t_S for which the value of q was found must be used in equation (18) when S_1 is computed. The value of S_1 thus obtained is then used in equation (18) with the required values of b_S/t_S and l_S/t_S to obtain the desired values of q . This value of q may then be used in equation (13) to compute β' . A similar scheme is used to find the value of c . That is, the value of c found in the table corresponding to the value of the b_W/t_S

under consideration and the highest value of b_S/t_S appearing for that value of b_W/t_S , together with the value of S_1 previously obtained, are used in equation (19) to compute S_2 . The values of S_1 , S_2 , and the dimension ratios being considered are then used in equation (19) to compute c .

The ranges of the dimension ratios covered by the table and curves are:

Ratio	Range
Rib thickness/Skin thickness, t_W/t_S	0 to 4
Fillet radius/Skin thickness, r_W/t_S	0 to 16
Length of repeating elements/Skin thickness, b_S/t_S	0 to ∞
Rib height/Skin thickness, b_W/t_S	0 to ∞

An additional result incidental to the evaluation of q and c is that, for values of $r_W/t_S > 1$, no shear stress concentration exists in the cross section. (Stress-concentration factors were based on an average shear stress in the skin at a remote unaffected distance from the juncture of the skin and ribs.) No investigation of the stress concentration when $r_W/t_S < 1$ was pursued.

Illustrative Examples

In order to illustrate the method of obtaining the effectiveness coefficients and their significance, the effectiveness coefficients j , q , and c are calculated for the rib proportions used in the tests of reference 3 and one variation of that shape.

The dimensions of the rib cross section used in reference 3 are:

$$b_S = 1.00 \text{ in.} \quad t_S = 0.05 \text{ in.}$$

$$t_W = 0.10 \text{ in.} \quad b_W = 0.20 \text{ in.} \quad r_W = 0.10 \text{ in.}$$

The dimension ratios are then

$$t_W/t_S = 2$$

$$b_S/t_S = 20$$

$$b_W/t_S = 4$$

$$r_W/t_S = 2$$

These dimension ratios are considered in the first example. The second example considers the same dimension ratios except that the value of r_W/t_S is changed from 2 to 0.

Example 1.- From figure 6(b), for $\frac{t_W}{t_S} = 2$ and $\frac{r_W}{t_S} = 2$,

$$\frac{\sqrt[4]{K'}}{t_S} = 2.2$$

Therefore,

$$K' = 23.4 t_S^4$$

From equation (27) the torsion constant K is determined as

$$K = 23.4 t_S^4 + \frac{1}{3}(14 - 2)t_S^4 + \frac{1}{3}(2 - 2)(2)^3 t_S^4 = 27.4 t_S^4$$

The effectiveness coefficient j can now be computed from equation (15) as

$$j = \frac{27.4 t_S^4}{2(20)t_S^4} = 0.685$$

The value of b_S/t_S is larger than the largest value of b_S/t_S which appears under $t_W/t_S = 2$ and $r_W/t_S = 2$ in the table, and b_W/t_S is equal to the largest value of b_W/t_S which appears under $t_W/t_S = 2$ and $r_W/t_S = 2$. Figures 7 and 8 are therefore used to obtain values for q and c . From figure 7, the value of $\frac{b_S/t_S}{q} - 2 \frac{l_S}{t_S}$ for $r_W/t_S = 2$ and $t_W/t_S = 2$ is found to be 4.0. Hence

$$\frac{20}{q} - 14 = 4$$

or

$$q = 1.111$$

From figure 8, $\frac{b_S}{t_S} c$ is found to be 2.10, or

$$c = 0.105$$

Using the above values of j , q , and c in equations (6), (7), and (8) gives the values of D_k , G_k , and C_k as follows:

$$D_k = 42.8 \times 10^{-6} G$$

$$G_k = 0.0555 G$$

$$C_k = -0.00525$$

The value of α' is found from equation (12) by substitution of the values of j and q previously found:

$$\alpha' = \frac{1}{5} \sqrt{\frac{1}{2} \left(\frac{1.111}{1.111 - 1} \right) (0.685 - 0.166)} = 0.3225$$

From equation (14),

$$\frac{A_W}{b_S t_S} = \frac{1}{20} \left[4(2) + 2 \left(1 - \frac{\pi}{4} \right) (2)^2 \right] = 0.486$$

The value of β' is then found from equation (13) as

$$\beta' = \frac{1.111 - 1}{0.486} = 0.2285$$

Example 2.- In this example, a configuration having the following proportions is considered:

$$t_W/t_S = 2 \quad b_S/t_S = 20$$

$$b_W/t_S = 4 \quad r_W/t_S = 0$$

The value of j may be found by means of equation (24) or from figure 6(b) and equation (15) to be

$$j = 0.436$$

From figures 7 and 8, q and c are found to be

$$q = 1.031$$

and

$$c = 0.032$$

The values of D_k , G_k , and C_k are then found from equations (6), (7), and (8) to be

$$D_k = 27.30 \times 10^{-6} G$$

$$G_k = 0.05155 G$$

$$C_k = - 0.00160$$

From equation (12),

$$\alpha' = 0.424$$

From equation (14),

$$\frac{A_W}{b_S t_S} = 0.40$$

The value of β' is then found from equation (13) to be

$$\beta' = 0.0775$$

The values of α' and β' found in these two examples could have been used in the formulas of reference 4 to obtain the elastic constants D_k , G_k , and C_k . The values of D_k and G_k so obtained would be exactly those values obtained by using the effectiveness coefficients j and q in equations (6) and (7). The values of C_k obtained from the formulas of reference 4, however, are found to be somewhat greater than those computed directly by use of the coefficient c .

This discrepancy, which disappears when ribs of small twisting stiffness are considered, arises as a result of the assumption of reference 4 that the shearing effectiveness of the ribs can be represented by a substitute sheet of zero twisting stiffness. There is no corresponding discrepancy in the calculation of D_k , however, since in reference 4 the location of the substitute sheet (measured by $\alpha'H$) is chosen to give the correct value of twisting stiffness for the stiffened plate as a whole. If a value of α' is desired which will give the correct value of the coupling term C_k , that value of α' may be obtained by equating the expression of reference 4 for C_k to that of the present paper and solving for α' . That procedure leads to the following expression for α' :

$$\alpha' = \frac{t_S}{H} \left(\frac{q}{q-1} \right) c \quad (29)$$

This value of α' would give correct values of C_k but somewhat conservative values of D_k .

Discussion of illustrative examples.— The most significant implication of the results of this evaluation of the effectiveness coefficients is that relatively small changes in detailed proportions can appreciably affect the overall effectiveness of integrally stiffened plates. As demonstrated in the examples, a change in fillet radius from $r_W/t_S = 2$ to $r_W/t_S = 0$ decreased β' (β' is a parameter which shows the efficiency of the rib in resisting shear) by a factor of 0.339 and decreased the twisting stiffness by a factor of 0.687.

More complete analysis will be required to evaluate fully the merits of large fillet radii and the effect of changes in the other proportions. Results of buckling tests of plates with fairly large fillet radii, such as those of reference 3, should not be considered representative of the results to be expected for similar plates with small fillet radii.

Suitably proportioned longitudinal or transverse integral stiffening can evidently contribute to the shear stiffness of plates; thus integral stiffeners may be utilized to contribute to the torsional stiffness as well as to the flexural stiffness of wing panels.

CONCLUDING REMARKS

The evaluation of the shearing effectiveness of integral stiffening for a wide range of proportions of rectangular stiffeners with circular fillets has indicated that the degree of penetration of stresses from the skin into the stiffeners is in part dependent upon the fillet radius. Also, for fillet radii greater than the skin thickness, the shear-stress-concentration factor has been found to be equal to unity. Determination of the overall structural importance of the fillet radius and the effect of changing other proportions require and, on the basis of the large changes in stiffness associated with small changes in configuration shown in the present study, deserve further investigation.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., March 3, 1955.

APPENDIX

ANALYSIS OF SHEARING EFFECTIVENESS OF LONGITUDINAL
OR TRANSVERSE INTEGRAL STIFFENING

Figure 3 shows the repeating element of a plate with integral unidirectional stiffeners. A quasi-shear state of deformation has been imposed upon the element so that the edge $\overline{af}$ has undergone a pure shear translation with respect to the edge $\overline{bc}$. The problem is to determine the stresses necessary to maintain the imposed deformation. The ratio of the resultants of the stresses to the magnitude of the assumed distortion provides an index to the effectiveness of integrally stiffened plates in resisting shearing forces.

Derivation of Differential Equation Governing Stress Function

The semi-inverse method of Saint-Venant, as found in reference 5 (pp. 259-263), is the approach used for this problem. Plane sections parallel to the yz -plane before distortion are assumed to have their shape preserved, but these planes may warp in the x -direction. This warping is the same for all cross sections along the x -axis. Displacements u in the x -direction of points in cross sections parallel to the yz -plane can be defined by a warping function,

$$u = \psi(y, z) \quad (A1)$$

Since the shape of the cross section is preserved, the displacements in the y - and z -directions (v and w), respectively, are

$$v = w = 0 \quad (A2)$$

The components of strain are therefore calculated from the relations between strains and displacements as

$$\left. \begin{aligned} \epsilon_x = \epsilon_y = \epsilon_z = \gamma_{yz} &= 0 \\ \gamma_{xz} &= \frac{\partial \psi}{\partial z} \\ \gamma_{xy} &= \frac{\partial \psi}{\partial y} \end{aligned} \right\} \quad (A3)$$

The corresponding stresses can then be calculated as

$$\left. \begin{aligned} \sigma_x = \sigma_y = \sigma_z = \tau_{yz} &= 0 \\ \tau_{xz} &= G \frac{\partial \psi}{\partial z} \\ \tau_{xy} &= G \frac{\partial \psi}{\partial y} \end{aligned} \right\} \quad (A4)$$

Consider now a stress function $\phi = \phi(y, z)$ from which the shear stresses τ_{xz} and τ_{xy} are obtainable. The equations of equilibrium given on page 229 of reference 5 must be satisfied. Only one of the three equations of equilibrium is of significance, namely,

$$\frac{\partial \tau_{xz}}{\partial z} + \frac{\partial \tau_{xy}}{\partial y} = 0 \quad (A5)$$

The stresses τ_{xz} and τ_{xy} may then be expressed in terms of the stress function $\phi = \phi(y, z)$; thus

$$\left. \begin{aligned} \tau_{xz} &= \frac{\partial \phi}{\partial y} \\ \tau_{xy} &= -\frac{\partial \phi}{\partial z} \end{aligned} \right\} \quad (A6)$$

Equating the stresses determined in equations (A4) to those determined in equations (A6), so as to determine the stresses from a consideration of displacements and thereby automatically satisfy compatibility of strains, yields

$$\left. \begin{aligned} G \frac{\partial \psi}{\partial z} &= \frac{\partial \phi}{\partial y} \\ G \frac{\partial \psi}{\partial y} &= -\frac{\partial \phi}{\partial z} \end{aligned} \right\} \quad (A7)$$

The warping function ψ may be eliminated from equations (A7) by differentiating both sides of the first of equations (A7) with respect to y and both sides of the second with respect to z and subtracting the second from the first. Elimination of the warping function shows that the stress function ϕ must satisfy Laplace's differential equation

$$\frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad (A8)$$

Stresses obtained from the solution of this differential equation satisfy the conditions of equilibrium and compatibility.

Determination of Boundary Conditions

The stresses normal to the boundaries $\overline{ab}$ and $\overline{cdef}$ of the element must be zero because these boundaries represent the stress-free surfaces of the repeating element. (See fig. 3.) The values of the stress function along these two boundaries must therefore be constant in order that the stresses normal to these boundaries may be zero. Boundary $\overline{ab}$ is arbitrarily set at the constant value of zero, and boundary $\overline{cdef}$ is arbitrarily set at the constant value of P . The physical significance of this choice of boundary conditions may be seen by considering the integrally stiffened plate to be a flat plate. Then,

$$\phi = \frac{P}{t_S} z \quad (A9)$$

is the solution to equation (A8). Thus,

$$\tau_{xy} = -\frac{P}{t_S} \quad (A10)$$

or

$$N_{xy} = \tau_{xy} t_S = -P \quad (A11)$$

Therefore P is the magnitude of the applied shear force per unit length.

Along the boundaries $\overline{af}$ and $\overline{bc}$, the displacements u are assumed to be constant. The shear strain γ_{xz} along those boundaries is given by

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial y} = 0 \quad (A12)$$

The shear stress τ_{xz} along those boundaries is therefore calculated to be zero, or

$$\frac{\partial \phi}{\partial y} = \tau_{xz} = 0 \quad (A13)$$

along boundaries $\overline{af}$ and $\overline{bc}$. The assumption that $\tau_{xz} = 0$ along boundaries $\overline{af}$ and $\overline{bc}$ could have been made from consideration of

the fact that the boundaries are lines of symmetry for the stress function, and therefore $\partial\phi/\partial y = 0$ at these lines of symmetry.

The problem has now been reduced mathematically to solving equation (A8) subject to the boundary conditions:

$$\left. \begin{aligned} \phi &= 0 \quad \text{along} \quad \overline{ab} \\ \phi &= P \quad \text{along} \quad \overline{cdef} \\ \frac{\partial\phi}{\partial y} &= 0 \quad \text{along} \quad \overline{af} \quad \text{and} \quad \overline{bc} \end{aligned} \right\} \quad (A14)$$

which are included in figure 2.

Determination of Resultant Forces

The resultant forces (see fig. 4) necessary to maintain the assumed state of deformation can now be determined in terms of the stress function $\phi = \phi(y, z)$. When these forces and the distortions produced by them are determined, the elastic constants can be obtained in terms of the stress function.

In the plane $y = b_S/2$, the shear stresses may be resolved to a resultant shearing force F_{yx} per unit length, acting at the midplane of the skin, and a torque T_{yx} per unit length. Equations (A6) and the boundary conditions (eqs. (A14)) are used to determine the forces in the plane $y = b_S/2$ as follows:

$$F_{yx} = \int_0^{t_S} (\tau_{xy})_{y=b_S/2} dz = -P \quad (A15)$$

and

$$\begin{aligned} T_{yx} &= \int_0^{t_S} (\tau_{xy})_{y=b_S/2} \left(\frac{t_S}{2} - z \right) dz \\ &= Pt_S \left(\frac{1}{2} - \frac{1}{Pt_S} \int_0^{t_S} \phi_{y=b_S/2} dz \right) \end{aligned} \quad (A16)$$

In a like manner, the resultant forces on the yz -plane are obtained as

$$F_{xy} = \frac{1}{b_S} \int_{-b_S/2}^{b_S/2} \int_0^h \tau_{xy} dz dy = -P \quad (A17)$$

and

$$\begin{aligned} T_{xy} &= \frac{1}{b_S} \int_{-b_S/2}^{b_S/2} \int_0^h \left[\tau_{xy} \left(\frac{t_S}{2} - z \right) + \tau_{xz} y \right] dz dy \\ &= P t_S \left(\frac{1}{2} + 2 \frac{A_W}{b_S t_S} - \frac{2}{P b_S t_S} \int_{-b_S/2}^{b_S/2} \int_0^h \phi dz dy + \frac{1}{P t_S} \int_0^{t_S} \phi_{y=b_S/2} dz \right) \end{aligned} \quad (A18)$$

where h is the z -distance from the y -axis to the boundary $\overline{cdef}$.

Derivation of Formula for q

The relative shear displacement δ of plane $\overline{af}$ at $y = b_S/2$ with respect to plane $\overline{bc}$ at $y = -b_S/2$ at any value of z between 0 and t_S is given by

$$\delta = \int_{-b_S/2}^{b_S/2} \gamma_{xy} dy \quad (0 \leq z \leq t_S) \quad (A19)$$

From this equation, equations (A6), and the stress-strain relationship $\tau_{xy} = G\gamma_{xy}$, δ becomes

$$\delta = -\frac{1}{Gz} \int_0^z \int_{-b_S/2}^{b_S/2} \frac{\partial \phi}{\partial z} dy dz \quad (0 \leq z \leq t_S) \quad (A20)$$

The average shear strain over the length of the repeating element is given by

$$\bar{\gamma}_{xy} = \frac{\delta}{b_S} = -\frac{1}{G b_S t_S} \int_{-b_S/2}^{b_S/2} \phi_{z=t_S} dy \quad (A21)$$

From equation (A17), $F_{xy} = F_{yx} = -P$; therefore, each of these two shear forces can be replaced by the more conventional notation N_{xy} . The coefficient q , which represents the effectiveness of the integral stiffener in resisting shear, can now be defined as follows:

$$q = \frac{N_{xy}}{G t_S \bar{\gamma}_{xy}} \quad (A22)$$

Substitution from equation (A15) or (A17) for N_{xy} and from equation (A21) for $\bar{\gamma}_{xy}$ in equation (A22) yields the coefficient q in the following form:

$$q = \frac{b_S}{\frac{1}{P} \int_{-b_S/2}^{b_S/2} \phi_{z=t_S} dy} \quad (A23)$$

Noting that $\phi = P$ when $z = t_S$ and $r_W + \frac{t_W}{2} \leq |y| \leq l_S + r_W + \frac{t_W}{2}$ (see fig. 2) and expressing the dimensions as dimensionless ratios permit q to be obtained in the form

$$q = \frac{b_S/t_S}{2 \frac{l_S}{t_S} + \frac{1}{P t_S} \int_{-\left(r_W + \frac{t_W}{2}\right)}^{r_W + \frac{t_W}{2}} \phi_{z=t_S} dy} \quad (A24)$$

Derivation of Formula for c

The resultant torques T_{xy} and T_{yx} necessary to maintain the assumed state of deformation are defined in terms of two coefficients $c_{T_{xy}}$ and $c_{T_{yx}}$ as

$$T_{xy} = -c_{T_{xy}} t_S N_{xy} \quad (A25)$$

$$T_{yx} = -c_{T_{yx}} t_S N_{xy} \quad (A26)$$

The torques are negative if N_{xy} is positive. This can be seen by comparing F_{yx} of equation (A15) and T_{yx} of equation (A16). From equations (A17), (A18), and (A25), $c_{T_{xy}}$ is obtained as

$$c_{T_{xy}} = \frac{1}{2} + 2 \frac{A_W}{b_{stS}} - \frac{2}{P_{bstS}} \int_{-b_S/2}^{b_S/2} \int_0^h \phi \, dz \, dy + \frac{1}{P_{tS}} \int_0^{t_S} \phi_{y=b_S/2} dz \quad (A27)$$

Similarly, $c_{T_{yx}}$ is obtained from equations (A17), (A18), and (A26) as

$$c_{T_{yx}} = \frac{1}{2} - \frac{1}{P_{tS}} \int_0^{t_S} \phi_{y=b_S/2} dz \quad (A28)$$

In order that these results may be incorporated in flat-plate theory, the twisting moments on adjacent sides of a repeating element must be equivalent. The distributed moments T_{xy} and T_{yx} may be replaced by concentrated lateral forces T_{xy} and T_{yx} at the corners of the plate as is done in reference 11. The resultant torques T_{xy} and T_{yx} are then replaced by M_{xy} where

$$M_{xy} = \frac{1}{2}(T_{xy} + T_{yx}) \quad (A29)$$

This system is statically equivalent to the actual system. It therefore produces essentially the same distortions as the actual system except in regions at the edges of the plate comparable in width to the thickness of the plate. This result follows from Saint-Venant's principle (see ref. 5, p. 33). The resultant torque M_{xy} is therefore defined in terms of a coefficient c as

$$M_{xy} = -ct_S N_{xy} \quad (A30)$$

where

$$\begin{aligned} c &= \frac{1}{2}(c_{T_{xy}} + c_{T_{yx}}) \\ &= \frac{1}{2} + \frac{A_W}{b_{stS}} - \frac{1}{P_{bstS}} \int_{-b_S/2}^{b_S/2} \int_0^h \phi \, dz \, dy \end{aligned} \quad (A31)$$

From equation (A20) the following relationship exists:

$$\frac{1}{t_S} \int_{-b_S/2}^{b_S/2} \phi_{z=t_S} dy = \frac{1}{z} \int_{-b_S/2}^{b_S/2} \phi_{z=z} dy \quad (0 \leq z \leq t_S) \quad (A32)$$

Rearranging equation (A32) and integrating over the thickness of the skin yields

$$\int_0^{t_S} \int_{-b_S/2}^{b_S/2} \phi \, dy \, dz = \frac{t_S}{2} \int_{-b_S/2}^{b_S/2} \phi_{z=t_S} dy \quad (A33)$$

Thus, in finding the value of a double integral of the ϕ -surface over the area of the element, only the integral over the attached stiffener need be found, since the integral over the flat-sheet part of the element is found from just the line integral along the line $z = t_S$ over the length of the element.

Substituting equation (A33) into equation (A31), noting that $\phi = P$ when $z = t_S$ and $r_W + \frac{t_W}{2} \leq |y| \leq l_S + r_W + \frac{t_W}{2}$, and nondimensionalizing yields

$$c = \frac{\frac{r_W}{t_S} + \frac{1}{2} \frac{t_W}{t_S} + \frac{b_W}{t_S} \frac{t_W}{t_S} + 2 \left(1 - \frac{\pi}{4}\right) \left(\frac{r_W}{t_S}\right)^2 - \frac{1}{Pt_S^2} \int_{-\left(r_W + \frac{t_W}{2}\right)}^{r_W + \frac{t_W}{2}} \int_{t_S}^h \phi \, dz \, dy - \frac{1}{2Pt_S} \int_{-\left(r_W + \frac{t_W}{2}\right)}^{r_W + \frac{t_W}{2}} \phi_{z=t_S} dy}{b_S/t_S} \quad (A34)$$

Thus, the forces required to maintain the assumed state of deformation are known in terms of two integrals:

$$S_1 = \frac{1}{Pt_S} \int_{-\left(r_W + \frac{t_W}{2}\right)}^{r_W + \frac{t_W}{2}} \phi_{z=t_S} dy \quad (A35)$$

and

$$S_2 = \frac{1}{Pt_S^2} \int_{-\left(r_W + \frac{t_W}{2}\right)}^{r_W + \frac{t_W}{2}} \int_{t_S}^h \phi \, dz \, dy \quad (A36)$$

These two integrals are evaluated for the cases considered herein by means of an electrical analog computer.

Determination of Elastic Constants D_K , G_K , and C_K in

Terms of j , q , and c

The elastic constant D_K is defined in terms of the coefficient j in equation (6) where j is obtained by the methods discussed in the main text.

The state of deformation assumed in the determination of the coefficients q and c is characterized by

$$\frac{\partial^2 w}{\partial x \partial y} = 0 \quad (A37)$$

Thus, by setting the twist equal to zero in equation (1) and substituting from equation (A30) for M_{xy} , the elastic constant C_k is obtained as

$$C_k = -ct_s \quad (A38)$$

When the twist $\frac{\partial^2 w}{\partial x \partial y}$ is set equal to zero in equation (2) and $\bar{\gamma}_{xy}$ as may be determined from equation (A22) is substituted for γ_{xy} of equation (2), the elastic constant G_k is obtained as

$$G_k = Gjt_s \quad (A39)$$

From the relationships of equations (5) the remaining elastic constants D_{xy} , G_{xy} , and T become

$$D_{xy} = Gjt_s^3 \quad (A40)$$

$$G_{xy} = \frac{Gjqt_s}{2c^2q + j} \quad (A41)$$

$$T = \frac{c}{Gjt_s^2} \quad (A42)$$

Determination of α' and β' in Terms of j and q

Reference 4 derived expressions for the elastic constants of integrally stiffened plates in which the effectiveness of the ribs for resisting twisting and shearing is expressed in terms of two parameters α' and β' . The coefficient β' represents the part of the rib which is effective in resisting shear when this part is considered to be flattened out over the length of the element, thus increasing the effective flat-plate thickness of the element. The relation between β' and the applied shearing force N_{xy} is then

$$N_{xy} = G \left(t_S + \beta' \frac{A_W}{b_S} \right) \bar{\gamma}_{xy} \quad (A43)$$

Equating N_{xy} as obtained from equation (A22) to that as obtained in equation (A43) results in the following relationship between β' and q :

$$\beta' = \frac{q - 1}{A_W/b_S t_S} \quad (A44)$$

The coefficient α' represents the height above the midplane of the skin at which the centroid of the distributed fractional area of the rib is located to produce the required twisting stiffness. By substituting from equation (A43) for β' and from equation (9) for D_{xy} in equation (93) of reference 4, α' is determined in terms of j and q :

$$\alpha' = \frac{t_S}{H} \sqrt{\frac{1}{2} \left(\frac{q}{q - 1} \right) \left(j - \frac{1}{6} \right)} \quad (A45)$$

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TABLE I.- VALUES OF q , c , AND β' AS DETERMINED BY ANALOG COMPUTER

$\frac{r_W}{t_S}$	$\frac{b_S}{t_S}$	$\frac{b_W}{t_S}$	q	c	β'
$t_W/t_S = 0$					
1	2	1	1.167	0.084	0.778
2	4	2	1.267	.145	.623
3	6	3	1.373	.219	.579
6	12	6	1.695	.496	.540
8	16	8	1.840	.645	.489
12	24	12	2.090	.955	.424
16	32	16	2.370	1.224	.399
$t_W/t_S = 1$					
0	7	1	1.023	0.018	0.126
	7	2	1.023	.022	.080
	7	3	1.023	.024	.054
	5	1	1.033	.025	.165
	5	2	1.033	.026	.083
	5	3	1.033	.026	.055
	3	1	1.058	.043	.174
	3	2	1.058	.044	.087
	3	3	1.058	.045	.058
	1	1	1.240	.152	.240
	1	2	1.241	.156	.121
	1	3	1.242	.158	.081
1	9	1	1.070	.051	.441
	9	2	1.070	.053	.259
	9	3	1.071	.053	.186
	7	1	1.092	.064	.451
	7	2	1.094	.070	.274
	7	3	1.094	.070	.192
	5	1	1.138	.093	.486
	5	2	1.138	.097	.284
	5	3	1.138	.098	.201
	3	1	1.250	.143	.525
	3	2	1.250	.158	.309
	3	3	1.250	.160	.219
2	7	2	1.257	.165	.616
	7	3	1.257	.174	.459
	7	4	1.257	.174	.367
	5	2	1.401	.232	.687
	5	3	1.401	.245	.512
	5	4	1.401	.245	.408
3	9	3	1.343	.245	.451
	9	4	1.343	.246	.393
	9	5	1.343	.246	.348
	7	3	1.489	.316	.499
	7	4	1.489	.316	.435
	7	5	1.489	.316	.386
6	13	6	1.748	.565	.453
8	17	8	1.910	.716	.437
12	25	12	2.155	1.077	.392
16	33	16	2.420	1.361	.373

$\frac{r_W}{t_S}$	$\frac{b_S}{t_S}$	$\frac{b_W}{t_S}$	q	c	β'
$t_W/t_S = 2$					
0	8	1	1.076	0.063	0.304
	8	2	1.077	.075	.154
	8	3	1.078	.079	.104
	6	1	1.105	.085	.315
	6	2	1.106	.101	.159
	6	3	1.108	.104	.108
	4	1	1.166	.129	.332
	4	2	1.173	.153	.173
	4	3	1.173	.156	.115
	2	1	1.463	.285	.463
	2	2	1.500	.340	.250
	2	3	1.500	.342	.166
1	10	1	1.119	.093	.490
	10	2	1.123	.126	.278
	10	3	1.123	.131	.189
	8	1	1.157	.118	.517
	8	2	1.161	.158	.291
	8	3	1.161	.164	.200
	6	1	1.227	.159	.561
	6	2	1.231	.213	.313
	6	3	1.233	.220	.217
	4	1	1.409	.246	.673
	4	2	1.433	.331	.391
	4	3	1.440	.341	.270
2	10	2	1.245	.161	.462
	10	3	1.245	.164	.335
	10	4	1.245	.165	.263
	8	2	1.327	.203	.492
	8	3	1.327	.204	.358
	8	4	1.327	.207	.281
	6	2	1.489	.270	.551
	6	3	1.489	.273	.401
	6	4	1.489	.275	.315
4	12	4	1.502	.413	.405
	12	5	1.502	.413	.358
	12	6	1.502	.417	.319
	10	4	1.670	.503	.451
	10	5	1.670	.504	.397
	10	6	1.670	.506	.355
6	14	6	1.830	.654	.423
	14	7	1.830	.661	.395
	14	8	1.830	.663	.369
8	18	8	1.957	.840	.396
12	26	12	2.185	1.180	.359
16	34	16	2.455	1.543	.349

TABLE I.- VALUES OF q , c , AND β' AS DETERMINED BY ANALOG COMPUTER - Concluded

$\frac{r_W}{t_S}$	$\frac{b_S}{t_S}$	$\frac{b_W}{t_S}$	q	c	β'
$t_W/t_S = 3$					
0	{	9 1	1.126	0.107	0.379
		9 2	1.141	.164	.212
		9 3	1.143	.169	.143
		9 4	1.143	.171	.107
		7 1	1.172	.141	.401
		7 2	1.190	.212	.222
		7 3	1.192	.220	.150
		7 4	1.192	.222	.112
		5 1	1.263	.202	.438
		5 2	1.289	.299	.241
		5 3	1.295	.310	.164
		5 4	1.295	.321	.123
		3 1	1.604	.361	.604
		3 2	1.676	.518	.338
		3 3	1.714	.550	.238
		3 4	1.714	.558	.178
1	{	9 1	1.228	.166	.599
		9 2	1.248	.227	.347
		9 3	1.257	.251	.245
		9 4	1.259	.260	.188
		7 1	1.328	.218	.670
		7 2	1.349	.293	.380
		7 3	1.359	.324	.266
		7 4	1.360	.337	.203
		5 1	1.553	.331	.806
		5 2	1.582	.413	.453
		5 3	1.587	.454	.311
		5 4	1.590	.471	.237
2	{	11 2	1.302	.261	.430
		11 3	1.310	.289	.318
		11 4	1.311	.299	.249
		11 5	1.311	.301	.205
		9 2	1.395	.319	.460
		9 3	1.406	.352	.341
		9 4	1.408	.366	.268
		9 5	1.408	.369	.219
		7 2	1.573	.409	.519
		7 3	1.591	.452	.386
		7 4	1.595	.470	.303
		7 5	1.595	.475	.249
4	{	13 4	1.551	.490	.380
		13 5	1.557	.499	.331
		13 6	1.562	.505	.294
		13 7	1.564	.507	.263
		11 4	1.730	.580	.426
		11 5	1.732	.591	.368
		11 6	1.741	.596	.328
		11 7	1.743	.597	.294
6	{	15 6	1.899	.740	.403
		15 7	2.020	.750	.370
8	{	19 8	2.020	.930	.385
		19 9	2.020	.940	.377
12	27	12	2.250	1.290	.356
16	35	16	2.490	1.710	.330

$\frac{r_W}{t_S}$	$\frac{b_S}{t_S}$	$\frac{b_W}{t_S}$	q	c	β'
$t_W/t_S = 4$					
0	{	6 1	1.343	0.250	0.515
		6 2	1.400	.380	.300
		6 3	1.424	.456	.212
		6 4	1.426	.485	.160
		4 1	1.671	.393	.671
		4 2	1.852	.606	.426
		4 3	1.911	.718	.304
		4 4	1.917	.768	.229
2	{	10 2	1.509	.397	.524
		10 3	1.526	.461	.383
		10 4	1.529	.489	.299
		10 5	1.531	.506	.244
		8 2	1.729	.497	.600
		8 3	1.757	.576	.441
		8 4	1.765	.613	.345
		8 5	1.770	.635	.284
4	{	12 4	1.815	.682	.427
		12 5	1.816	.698	.365
		12 6	1.818	.713	.318
		12 7	1.820	.713	.282
6	{	16 6	1.950	.855	.386
		16 7	1.950	.860	.350
		16 8	1.950	.865	.320
		16 9	1.950	.865	.296
8	{	20 8	2.070	.995	.336
		20 9	2.070	1.002	.315
		20 10	2.070	1.006	.296
		20 11	2.070	1.007	.280
12	28	12	2.290	1.420	.330
16	36	16	2.540	1.880	.317

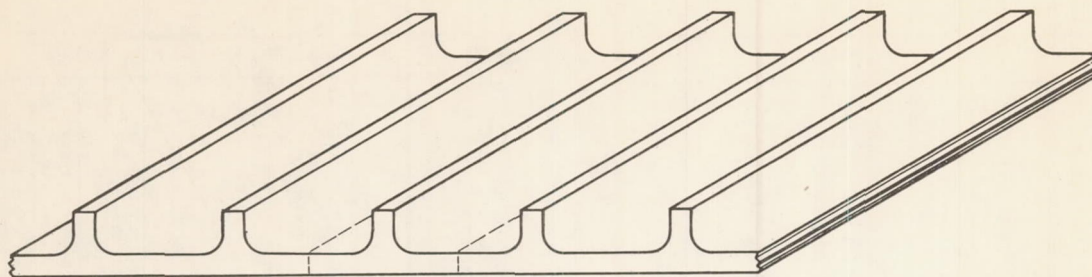


Figure 1.- Integrally stiffened plate considered.

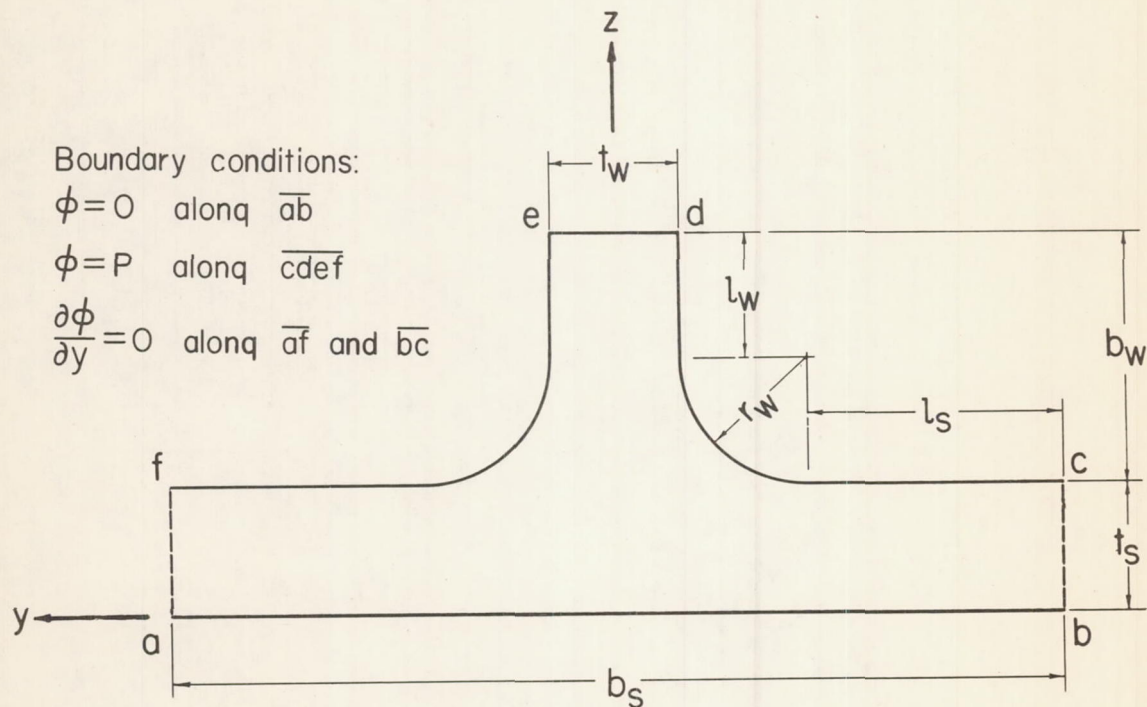


Figure 2.- Cross section of repeating element and boundary conditions on stress function ϕ .

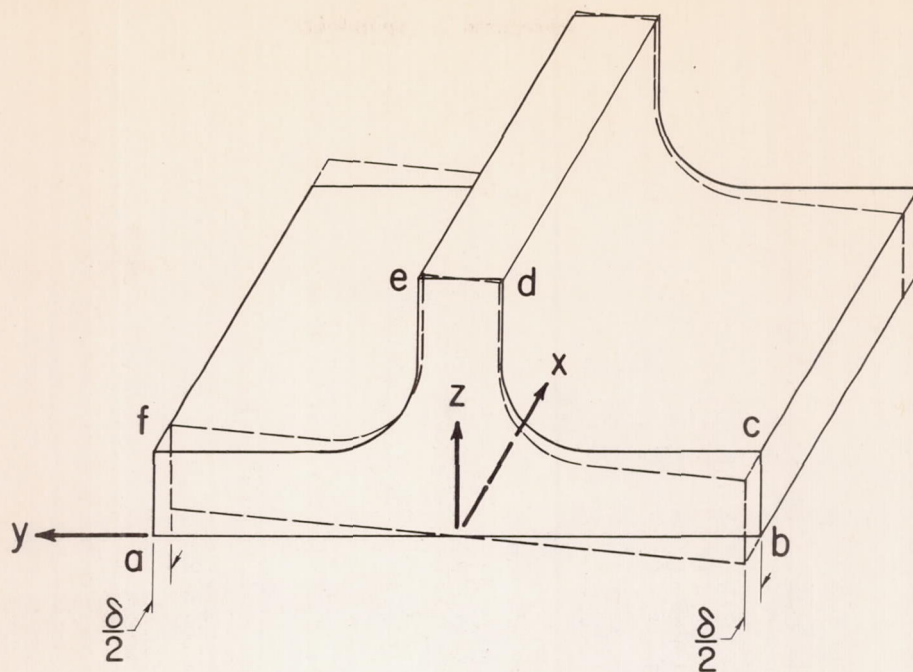


Figure 3.- Repeating element in imposed state of deformation.

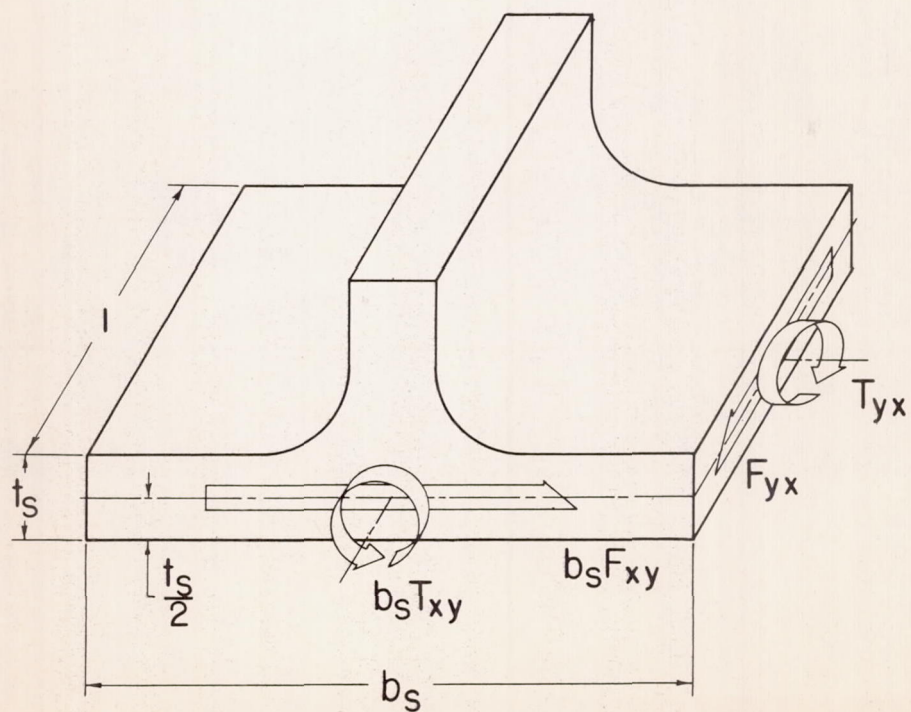


Figure 4.- Repeating element with resultant forces necessary to maintain imposed deformation.

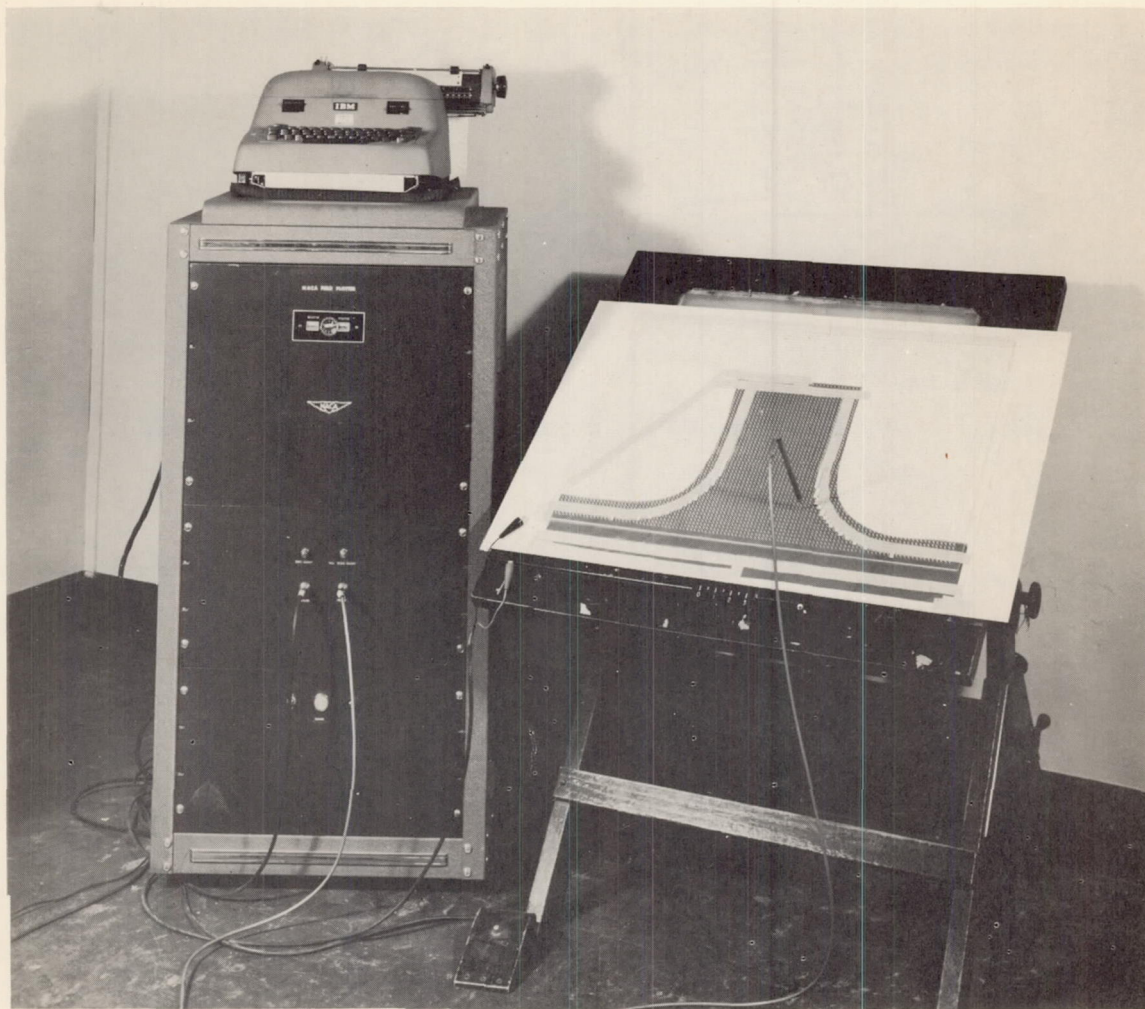
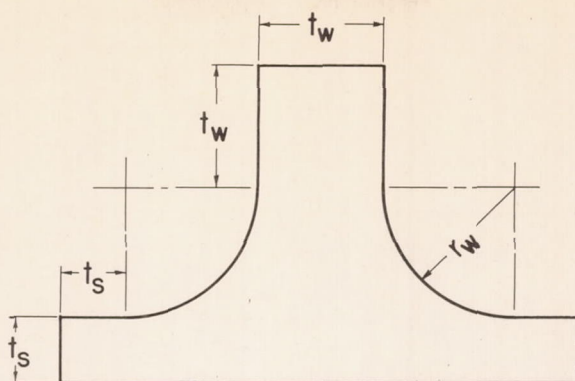


Figure 5.- Analog Field Plotter.

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(a) Cross section of repeating element.

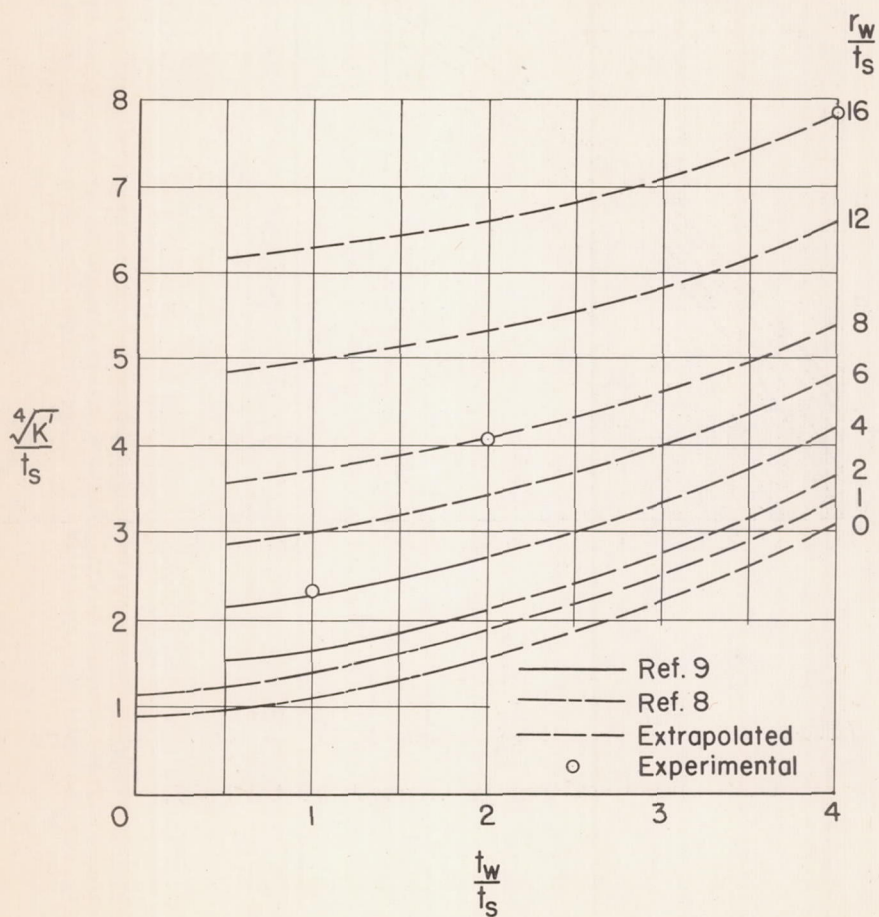
(b) Values of $\sqrt[4]{K'}/t_s$.

Figure 6.- Values of $\sqrt[4]{K'}/t_s$ for repeating element when
and $t_w = t_s$.

 t_s

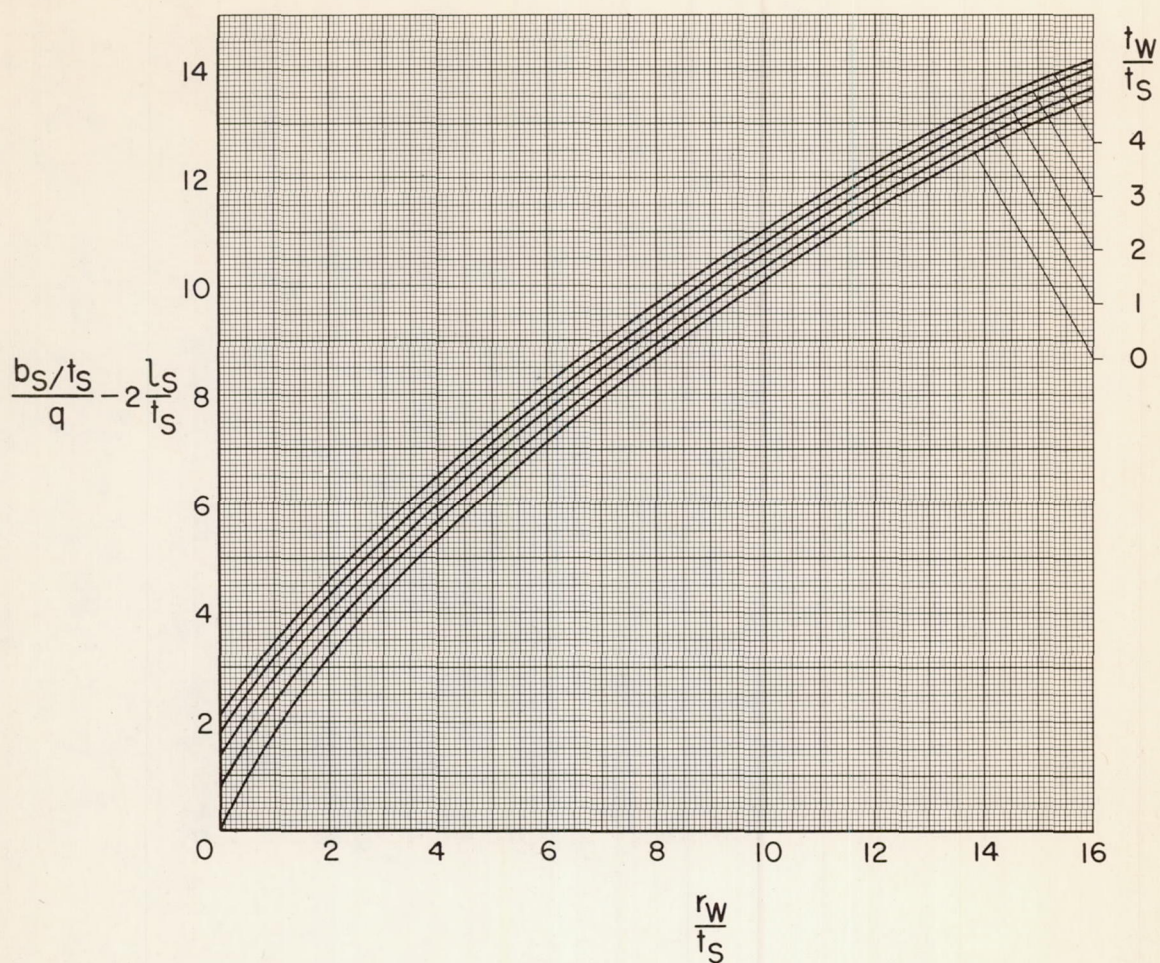


Figure 7.- Values of $\frac{b_s/t_s}{q} - 2 \frac{l_s}{t_s}$ when both $\frac{b_w}{t_s}$ and $\frac{b_s}{t_s}$ are larger than those values presented in table I.

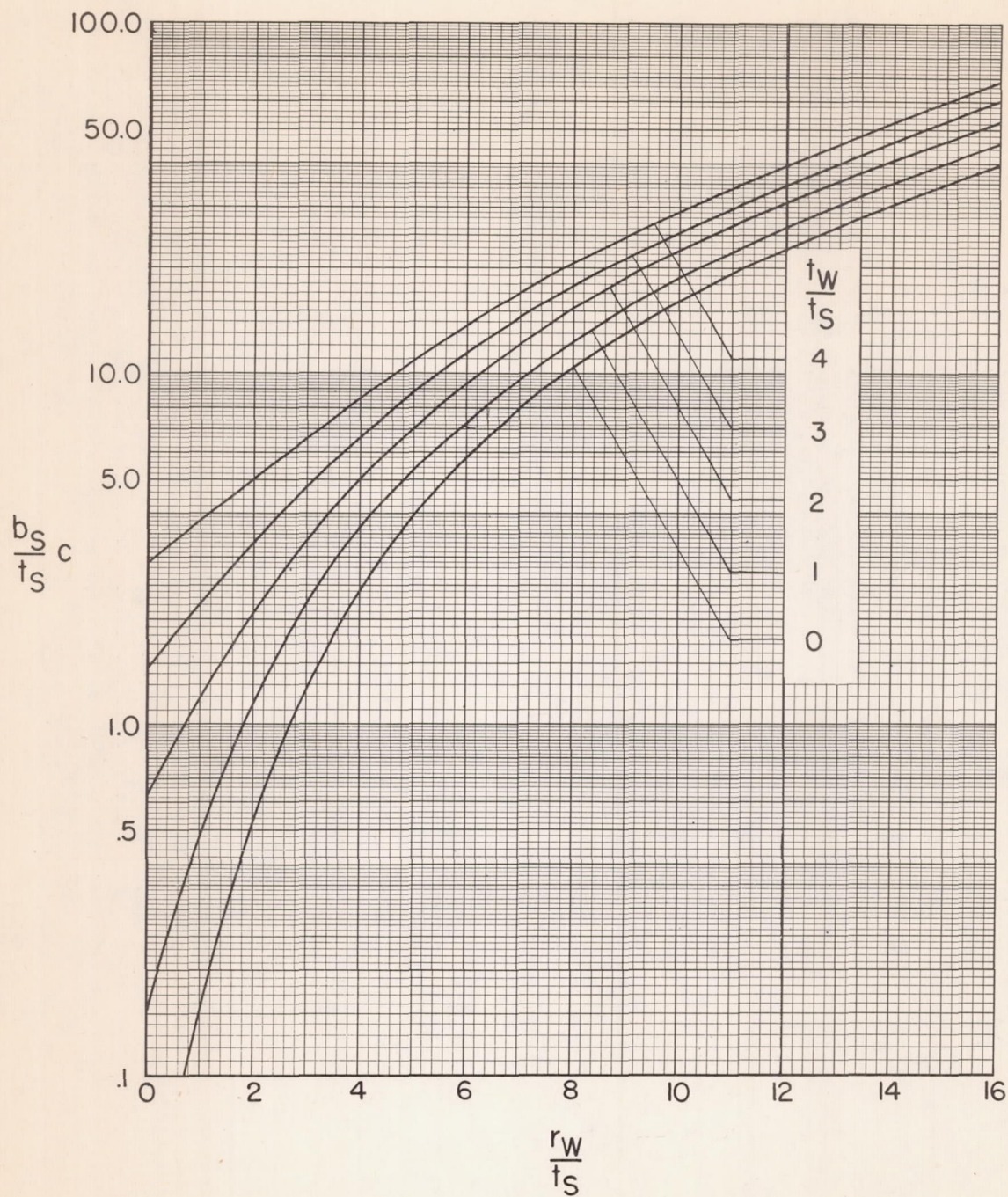


Figure 8.- Values of $\frac{b_S}{t_S} c$ when both $\frac{b_W}{t_S}$ and $\frac{b_S}{t_S}$ are larger than those values presented in table I.

